

Exercise 88

For the following exercises, find the composition when $f(x) = x^2 + 2$ for all $x \geq 0$ and $g(x) = \sqrt{x - 2}$.

$$(g \circ f)(a); \quad (f \circ g)(a)$$

Solution

Write $(f \circ g)(x)$.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= (\sqrt{x - 2})^2 + 2 \\ &= (x - 2) + 2 \\ &= x\end{aligned}$$

Write $(g \circ f)(x)$.

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= \sqrt{(x^2 + 2) - 2} \\ &= \sqrt{x^2} \\ &= |x| \\ &= x\end{aligned}$$

Therefore,

$$(g \circ f)(a) = a \quad \text{and} \quad (f \circ g)(a) = a.$$